SS 2012 9. Mai 2012

In-class Exercises to the Lecture Logics
Sheet 2

Jun.-Prof. Dr. Roland Meyer Discussion on 10./11. Mai 2012

Exercise 2.1 [Gentzen Calculus]

a) Present a definition of *soundness* for the Gentzen calculus.

b) Show that the Gentzen calculus is sound with respect to your definition.

c) Show that the axioms of the calculus $\mathcal{F}_0$ are derivable in the Gentzen calculus.

Exercise 2.2 [Completeness of $\mathcal{F}_0$]

Prove Lemma 1.24 on the slides: Let $A \equiv A(p_1, \dots, p_n) \in F$, $n > 0$, where $p_1, \dots, p_n$ are the propositional variables occurring in A. Let φ be a valuation. If

$$P_i := \begin{cases} p_i, & \text{if } \varphi(p_i) = 1, \\ \neg p_i, & \text{if } \varphi(p_i) = 0, \end{cases} \qquad A' := \begin{cases} A, & \text{if } \varphi(A) = 1, \\ \neg A, & \text{if } \varphi(A) = 0, \end{cases}$$

for $1 \leqslant i \leqslant n$, then $P_1, \dots, P_n \vdash A'$.

Exercise 2.3 [Derivations in $\mathcal{F}_0$]

Present proofs in $\mathcal{F}_0$ for the following formulae:

a) $(A \to B) \to (\neg B \to \neg A)$. You can use the Deduction Theorem and the given theorems 1–7 from the slides.

b) $B \to (\neg C \to \neg(B \to C))$. You can use the Deduction Theorem and the given theorems 1–8 from the slides.

Exercise 2.4 [Completeness of calculi]

Let $\mathcal{K} = (\mathrm{Ax}, R)$ be a calculus, where R contains only Modus Ponens and Ax is given by just one axiom scheme, namely $(\neg A \to \neg B) \to (B \to A)$.

a) Show by induction on n that for each proof $B_0, \dots, B_n$ and each $i \in \{0, \dots, n\}$, the following holds: The number of occurrences of each propositional variable p in B_i is even.

b) Conclude that in $\mathcal{K}$, not every tautology is derivable (even if it contains only $\neg$ and $\to$ as connectives).