SS 2012 04.07.2012

In-class Exercises to the Lecture Logics
Sheet 6

Jun.-Prof. Dr. Roland Meyer Discussion on 05./06.07.2012

Exercise 6.1 [Tableaux for predicate logic]
Using tableaux, determine whether

a) the formula $\forall z \exists x \exists y p(x, y, z)$ is satisfiable.

b) the formula $\forall x \forall y \left((p(x, y) \wedge q(x)) \rightarrow \exists y q(y)\right)$ is a tautology.

Exercise 6.2 [Decidable theories]
Let T be a recursively decidable theory. Show that there is a recursively enumerable system of axioms that generates T.

Exercise 6.3 [A validity test]
Show that there is an algorithm that, given a formula of the form

$$\forall x_1 \cdots \forall x_n \exists y_1 \cdots \exists y_m B,$$

where B contains no function symbols, determines whether the formula is a tautology.

Exercise 6.4 [The Compactness Theorem for predicate logic]
Let A be a formula in first order predicate logic such that for each $n \in \mathbb{N}$, A has a model I with $|I| \geqslant n$. In Exercise 5.1, you have shown that for each $n \in \mathbb{N}$, there is a formula B_n with $I \models B_n$ if and only if $|I| \geqslant n$. Consider the set $\Sigma = \{A \wedge B_n \mid n \in \mathbb{N}\}$.

a) Using the compactness theorem for first order predicate logic, show that Σ is satisfiable.

b) Show that A has an infinite model.

c) Conclude that there is no formula F such that $I \models F$ if and only if I has a finite domain.