In-class Exercises to the Lecture Logics
Sheet 7

Jun.-Prof. Dr. Roland Meyer   Discussion on 19./20.07.2012

Exercise 7.1 [The Löwenheim-Skolem Theorem]

A variant of Herbrand's Theorem reads: A formula in first order predicate logic without "=" is satisfiable if and only if it has a Herbrand model.

a) Present a satisfiable formula in first order predicate logic that has no Herbrand model.

b) Show that for each formula in first order predicate logic, there is an equisatisfiable formula A' without "=".

c) Prove that for each formula in first order predicate logic, there is a formula A' without "=" with the following property: A is satisfiable if and only if it has a model that is obtained by a quotient set construction from a model of A'.

d) Conclude that each satisfiable formula in first order predicate logic has a countable model. *Note:* This result is also known as the Löwenheim-Skolem Theorem.

Exercise 7.2 [Resolution]

Consider the formula

$$A \equiv \forall x \left[p(x) \wedge (q(z,b) \rightarrow \exists y(\neg q(x,y) \vee \neg p(y))) \right] \wedge \forall x \forall y q(x,y).$$

a) Determine a formula equisatisfiable in clause form which is equisatisfiable to A.

b) Using the resolution calculus, show that A is unsatisfiable.