SS 2013 June 19th, 2013

In-class Exercises to the Lecture Logics
Sheet 5

Jun.-Prof. Dr. Roland Meyer Discussion on June 20th and 21st, 2013

Exercise 5.1 [Formulae in predicate logic]

a) Let $A \equiv \forall x \exists y p(x, y)$ and $B \equiv \exists y \forall x p(x, y)$. Which of these formulas is deducible from the other? Are they equivalent?
b) Is the formula $\forall x p(x) \to \exists x p(x)$ a tautology?

Exercise 5.2 [Tautologies]
Suppose A' is formula in predicate logic that is obtained from a formula A in propositional logic by replacing each variable with an atomic formula in predicate logic. Here, all occurrences of a given variable should be replaced by the same atomic formula. *Example:* If $A = (p \wedge q) \to (p \vee q)$, then A' could be $(r(a, b) \wedge s(c)) \to (r(a, b) \vee s(c))$.

Prove: If A is a tautology in propositional logic, then A' is a tautology in predicate logic.

Exercise 5.3 [Elimination of "="]
We write $\mathsf{FO}^{\neq}(S)$ for the set of formulae in predicate logic over the signature S in which the symbol "=" does not occur.

a) Devise a method that transforms a formula $A \in \mathsf{FO}(S)$ into an equisatisfiable formula $A' \in \mathsf{FO}^{\neq}(S)$.
b) Describe how, given a model for A', one can construct a model for A.

Exercise 5.4 [Skolem normal form]

a) Suppose $A \equiv \forall y_1 \cdots \forall y_n \exists z B$. Furthermore, let $f/n \in \mathsf{Sko}$ be a Skolem symbol not occurring in B. Show that

$$\forall y_1 \cdots \forall y_n B\{z/f(y_1, \ldots, y_n)\}$$

is equisatisfiable with A.
b) Conclude that the algorithm in Definition 3.26 yields an equisatisfiable formula.
c) Show that Skolemization can yield a formula that is not necessarily equivalent to the input formula. Consider, for example, the formula $\forall x \exists y p(x, y)$.