Theoretical Computer Science 1

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Exercise 2

TU Braunschweig

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Hand in your solutions per e-mail to your tutor until Thursday, 02.12.2021 23:59 pm. You should provide your solutions either directly as .pdf file or as a readable scan/photo of your handwritten notes. Submit your results as a group of four.

Exercise 1: Reaching Definitions [9 points]

Perform a Reaching-Definitions-Analysis on the following program.

$[x := 0]^0$

while $[x < 2^4]^1$ **do**

    $[y := x^7 + x^4 + x^3 + x^2 + 1 \bmod 2^{16}]^2$

    if $[y \bmod 3 \neq 0]^3$ **then**

        $[y := y + 1]^4$

    end if

    if $[y < 2^{15}]^5$ **then**

        break

    end if

    $[x := x + 1]^6$

end while

$[y := 2^4 - x]^7$

a) [1 point] Draw the control flow graph G for $B = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Mark its extremal blocks. Note that this is a forwards analysis.

b) [3 points] Consider the lattice $\mathcal{D} = \langle \mathcal{P}(\{x, y\} \times (B + \{?\})), \subseteq \rangle$. Assign for each block $b \in B$ a suitable, monotone transfer function f_b over this lattice.

c) [5 points] Consider the data flow system $\langle G, \mathcal{D}, \{(x, ?), (y, ?)\}, (f_b)_{b \in B} \rangle$. Write down the induced equation system and determine its least solution using Kleene's fixed point theorem.

Exercise 2: Live Variables [9 points]

Perform a Live-Variables-Analysis on the following program.

$[x := 0]^0$
while $[x^2 < y]^1$ **do**
$\quad |\quad [x := x + 1]^2$
end while
if $[x^2 = y]^3$ **then**
$\quad |\quad [x := 1]^4$
else
$\quad |\quad [x := 0]^5$
end if

a) [1 point] Draw the control flow graph G for $B = \{0, 1, 2, 3, 4, 5\}$ and mark its extremal blocks. Note that this is a backwards analysis.

b) [3 points] Consider the lattice $\mathcal{D} = \langle \mathcal{P}(\{x, y\}), \subseteq \rangle$. Assign for each block $b \in B$ a suitable, monotone transfer function f_b over this lattice.

c) [5 points] Consider the data flow system $\langle G, \mathcal{D}, \varnothing, (f_b)_{b \in B} \rangle$. Write down the induced equation system and determine its least solution using Kleene's fixed point theorem.

## Exercise 3: REG to NFA [10 points]

Stepwise construct a finite automaton for the regular expression $(c \cup a(b \cup ca)^*(\varepsilon \cup cc))((a \cup b)c^* \cup a^*b(cb)^*)^*$ over the alphabet $\{a, b, c\}$. It suffices to draw a state chart for each automaton.

Note: The operator precedence is as follows: $(^*)$, $(.)$, $(\cup)$

a) [1 point] Give an automaton A for $(a \cup b)c^*$.

b) [1 point] Give an automaton B for $a^*b(cb)^*$.

c) [2 points] Give an automaton C for $\mathcal{L}(A) \cup \mathcal{L}(B)$.

d) [2 points] Give an automaton D for $\mathcal{L}(C)^*$.

e) [2 points] Give an automaton E for $c \cup a(b \cup ca)^*(\varepsilon \cup cc)$. You will not need more than three states.

f) [2 points] Give an automaton F for $\mathcal{L}(E).\mathcal{L}(D)$.

Exercise 4: NFA to REG using Arden's Rule [7 points]

Consider the following NFA A over the alphabet $\{a, b\}$:
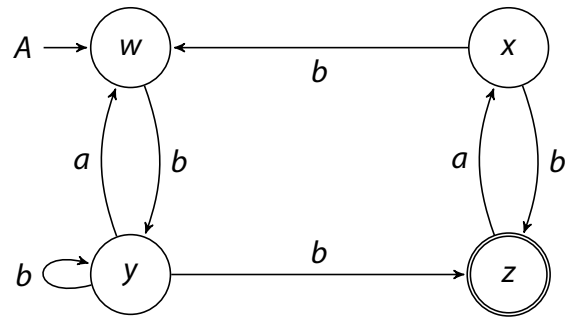
a) [1 point] Formulate the equation system associated with A.

b) [4 points] Find a regular expression for $\mathcal{L}(A)$ by solving the equation system using Arden's Rule.

c) [2 points] State the solution of the equation system for the state z. Give an NFA for this regular expression.