Memory Model-aware Testing

a Unified Complexity Analysis

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- Programmers expect sequential consistency.

Gibbons, Korach 1997
Cantin, Lipasti, Smith 2005

Introduction

Motivation

- Programmers expect sequential consistency.
- Modern architectures lack sequential consistency.

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- Weak memory models may introduce undesired states.

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- State explosion for reachability analysis.

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- Weak memory models may introduce undesired states.
- State explosion for reachability analysis.

- **Complexity of Testing?**

Gibbons, Korach 1997
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- Test: sequences of reads/writes for multiple processes.
- Reads are blocking.
- Memory variables initialized to 0.

EXAMPLE: Test $\mathcal{T}$

P1
(w, x, 1)
(w, x, 2)

P2
(r, x, 1)

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◀ :

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$$x: \quad 2$$

Serial View

- Processes observe operations in different orders (views).
- A **serial view** ◀ = $\mathit{SerialView}(\mathcal{O}, <)$ is a sequence of operations from $\mathcal{O}$ that respects some partial order $<$.
- Always read from last write.

Steinke, Nutt 2004

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- A Test $\mathcal{T}$ is **executable under sequential consistency** if:

$$\exists \quad \blacktriangleleft = \mathit{SerialView}(\mathcal{T}, <_{PO}).$$

Example $\blacktriangleleft$: $\quad (w, x, 1).(r, x, 1).(w, x, 2)$

Steinke, Nutt 2004

The Testing Problem

Testing Problem of model M:

Given test $\mathcal{T}$, is it **executable under** *model* **M***?*

Steinke, Nutt 2004

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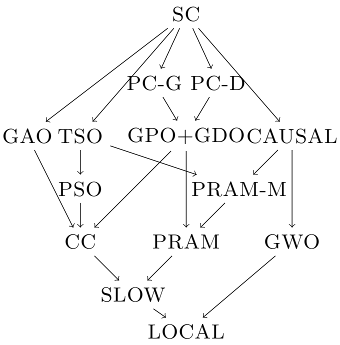

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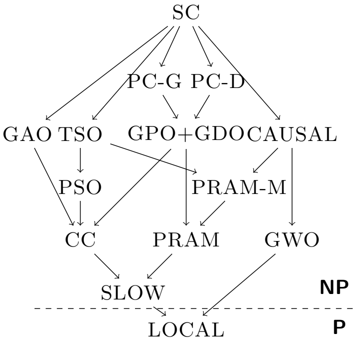

Steinke, Nutt 2004

The Testing Problem

- **Testing Problem is in NP for all models**
- Testing Problem is NP-hard for most models
- Testing Problem is in P for some models

Testing is in NP

Uniform Reduction to SAT:

- Formula:

  $$WT(\mathcal{T}) \wedge \mathrm{SV}_1 \wedge .. \wedge \mathrm{SV}_k$$

  WT: Unique Writes-To
  SV: SerialView properties

Testing is in NP

Uniform Reduction to SAT:

- Formula:

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  WT: Unique Writes-To
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- Boolean variable:

  $$sv_{i,j} \leftrightarrow (op_i \blacktriangleleft op_j)$$

- Serial view properties:

  Totality, Asymmetry, Transitivity, Read-Last-Write

The Testing Problem

- **Testing Problem is in NP for all models**
  - Uniform SAT reduction.
  - Optimal solution if NP-hard.
- Testing Problem is NP-hard for most models
- Testing Problem is in P for some models

The Testing Problem

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The Testing Problem

- Testing Problem is in NP for all models
- **Testing Problem is NP-hard for most models**
  - Our proofs cover multiple models
- Testing Problem is in P for some models

NP-hard for most models

Range reduction

$\mathbf{M}_{Strong} \preceq \mathbf{M}_{Weak}$**-range reduction** *$f$ of* SAT *to testing*:

$$
\begin{tikzcd}
 & & & \mathrm{M}_{Strong} \arrow[dl] \arrow[d] \arrow[dr] & \\
\mathrm{SAT} & & \mathrm{M} \arrow[r, dotted, no head] \arrow[dr] & \mathrm{M}' \arrow[r, dotted, no head] \arrow[d] & \mathrm{M}'' \arrow[dl] \\
 & & & \mathrm{M}_{Weak} &
\end{tikzcd}
$$

(i) ϕ is SAT $\Longrightarrow$ test $f(\phi)$ is executable under M_{Strong}.

(ii) test $f(\phi)$ is executable under M_{Weak} $\Longrightarrow$ ϕ is SAT.

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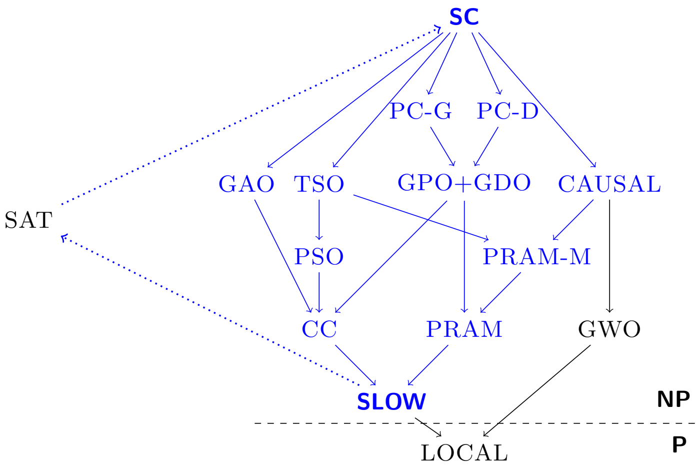
NP-hard for most models

SC ≤ SLOW-Range-Reduction



NP-hard for most models

Slow Consistency

Writes from one process to one variable are observed in same order by all processes.

Hutto, Ahamad 1990

NP-hard for most models

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- The program order is respected.
- For each process p and variable x: there exists a serial view on all writes to x and reads from x of p.

$$\forall x, p\ \exists \quad \blacktriangleleft = SerialView(\quad \mathcal{T}|_{w,x} \cup \mathcal{T}|_{p,x} \quad , <_{PO})$$

$$\exists \quad \blacktriangleleft = SerialView(\quad \mathcal{T} \quad , \qquad\qquad <_{PO}) \qquad\qquad [SC]$$

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NP-hard for most models

$\mathrm{SC} \leq \mathrm{SLOW}$-Range-Reduction of SAT

Reduction Idea

- Test uses only one variable ξ.
- Test has only one process with reads.
- $\Rightarrow$ Test behaves the same from Slow to SC.

$$\forall x, p \; \exists \quad \blacktriangleleft = \mathit{SerialView}(\; \mathcal{T}|_{w,x} \cup \mathcal{T}|_{p,x} \; , <_{PO}) \qquad [\mathit{Slow}]$$

$$\exists \quad \blacktriangleleft = \mathit{SerialView}(\; \mathcal{T} \; , <_{PO}) \qquad [\mathit{SC}]$$

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SC ≤ SLOW-Range-Reduction of SAT

Reduction Idea

- Test uses only one variable ξ.
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- $\Rightarrow$ Test behaves the same from Slow to SC.

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$$\exists\ \blacktriangleleft = SerialView(\ \mathcal{T}\ , <_{PO}) \qquad [SC]$$

SAT-Reduction

- We associate clauses and variables with values of ξ.

SC-Slow Reduction - Example

$$\underbrace{(a \vee b)}_{cl_1} \wedge \underbrace{\neg a}_{cl_2}$$

$a = false$
$b = true$

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$a = false$
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$$True_a := (w, \xi, cl1) \ . \ (w, \xi, a)$$
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Eval &:= (r, \xi, a) \ . \ (r, \xi, b) \ . \ (r, \xi, cl1) \ . \ (r, \xi, cl2)
\end{aligned}$$

◀ :

ξ : 0

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$\blacktriangleleft :\quad (w, \xi, cl1)$

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◀ : $(w, \xi, cl1).(w, \xi, a).(r, \xi, a).(w, \xi, b).(r, \xi, b)$
$.(w, \xi, cl1)$

ξ : 1

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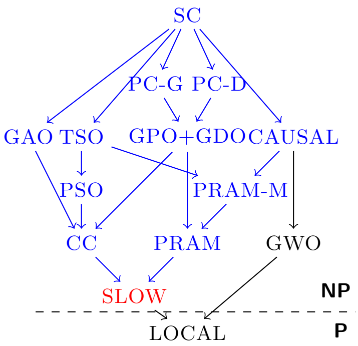
\end{aligned}$$

Results

Memory Model	Complexity Class of TEST(M)			
	General	Process	Length	Variables
SC	**NPC**(by 1)			**NPC**(by 1)
TSO	**NPC**(by 1)			**NPC**(by 1)
PSO	**NPC**(by 1)			**NPC**(by 1)
PC-G	**NPC**(by 1)			**NPC**(by 1)
PC-D	**NPC**(by 1)			**NPC**(by 1)
GAO	**NPC**(by 1)			**NPC**(by 1)
GPO+GDO	**NPC**(by 1)			**NPC**(by 1)
Causal	**NPC**(by 1)			**NPC**(by 1)
PRAM-M	**NPC**(by 1)			**NPC**(by 1)
GWO				
CC	**NPC**(by 1)			**NPC**(by 1)
PRAM	**NPC**(by 1)			**NPC**(by 1)
SLOW	**NPC**(by 1)			**NPC**$_1$
LOCAL				



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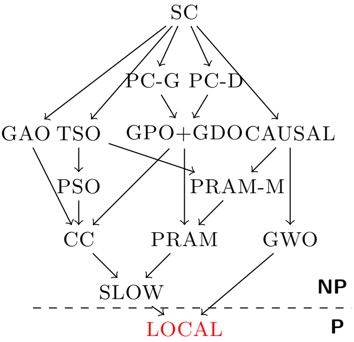
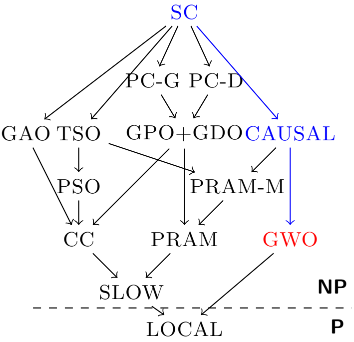
Memory Model	Complexity Class of TEST(M)			
	General	Process	Length	Variables
SC	**NPC**(by 1)			**NPC**(by 1)
TSO	**NPC**(by 1)			**NPC**(by 1)
PSO	**NPC**(by 1)			**NPC**(by 1)
PC-G	**NPC**(by 1)			**NPC**(by 1)
PC-D	**NPC**(by 1)			**NPC**(by 1)
GAO	**NPC**(by 1)			**NPC**(by 1)
GPO+GDO	**NPC**(by 1)			**NPC**(by 1)
Causal	**NPC**(by 1)			**NPC**(by 1)
PRAM-M	**NPC**(by 1)			**NPC**(by 1)
GWO				
CC	**NPC**(by 1)			**NPC**(by 1)
PRAM	**NPC**(by 1)			**NPC**(by 1)
SLOW	**NPC**(by 1)			$\mathbf{NPC}_1$
LOCAL	$\mathbf{P}_2$	**P**(by 2)	**P**(by 2)	**P**(by 2)



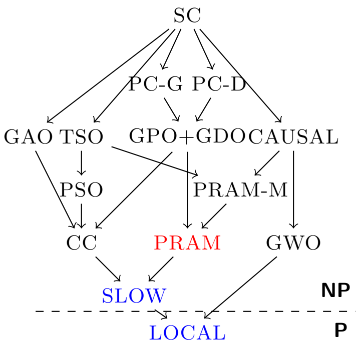
Results

Memory Model	Complexity Class of TEST(M)			
	General	Process	Length	Variables
SC	**NPC**(by 1)		**NPC**(by 3)	**NPC**(by 1)
TSO	**NPC**(by 1)			**NPC**(by 1)
PSO	**NPC**(by 1)			**NPC**(by 1)
PC-G	**NPC**(by 1)			**NPC**(by 1)
PC-D	**NPC**(by 1)			**NPC**(by 1)
GAO	**NPC**(by 1)			**NPC**(by 1)
GPO+GDO	**NPC**(by 1)			**NPC**(by 1)
Causal	**NPC**(by 1)		**NPC**(by 3)	**NPC**(by 1)
PRAM-M	**NPC**(by 1)			**NPC**(by 1)
GWO	**NPC**(by 3)		$\mathbf{NPC}_3$	
CC	**NPC**(by 1)			**NPC**(by 1)
PRAM	**NPC**(by 1)			**NPC**(by 1)
SLOW	**NPC**(by 1)			$\mathbf{NPC}_1$
LOCAL	$\mathbf{P}_2$	**P**(by 2)	**P**(by 2)	**P**(by 2)



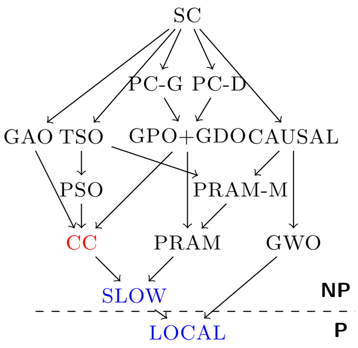
Results

Memory Model	Complexity Class of TEST(M)			
	General	Process	Length	Variables
SC	**NPC**(by 1)		**NPC**(by 3)	**NPC**(by 1)
TSO	**NPC**(by 1)			**NPC**(by 1)
PSO	**NPC**(by 1)			**NPC**(by 1)
PC-G	**NPC**(by 1)			**NPC**(by 1)
PC-D	**NPC**(by 1)			**NPC**(by 1)
GAO	**NPC**(by 1)			**NPC**(by 1)
GPO+GDO	**NPC**(by 1)			**NPC**(by 1)
Causal	**NPC**(by 1)		**NPC**(by 3)	**NPC**(by 1)
PRAM-M	**NPC**(by 1)			**NPC**(by 1)
GWO	**NPC**(by 3)		$\mathbf{NPC}_3$	
CC	**NPC**(by 1)			**NPC**(by 1)
PRAM	**NPC**(by 1)		$\mathbf{P}_4$	**NPC**(by 1)
SLOW	**NPC**(by 1)		**P**(by 4)	$\mathbf{NPC}_1$
LOCAL	$\mathbf{P}_2$	**P**(by 2)	**P**(by 2)	**P**(by 2)



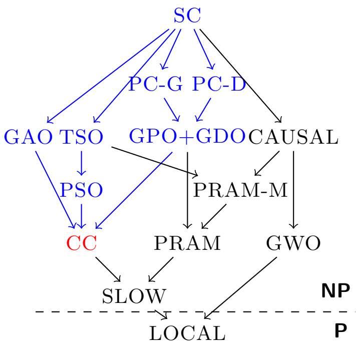
Results

Memory Model	Complexity Class of TEST(M)			
	General	Process	Length	Variables
SC	**NPC**(by 1)		**NPC**(by 3)	**NPC**(by 1)
TSO	**NPC**(by 1)			**NPC**(by 1)
PSO	**NPC**(by 1)			**NPC**(by 1)
PC-G	**NPC**(by 1)			**NPC**(by 1)
PC-D	**NPC**(by 1)			**NPC**(by 1)
GAO	**NPC**(by 1)			**NPC**(by 1)
GPO+GDO	**NPC**(by 1)			**NPC**(by 1)
Causal	**NPC**(by 1)		**NPC**(by 3)	**NPC**(by 1)
PRAM-M	**NPC**(by 1)			**NPC**(by 1)
GWO	**NPC**(by 3)		$\mathbf{NPC}_3$	
CC	**NPC**(by 1)	$\mathbf{P}_5$		**NPC**(by 1)
PRAM	**NPC**(by 1)		$\mathbf{P}_4$	**NPC**(by 1)
SLOW	**NPC**(by 1)	**P**(by 5)	**P**(by 4)	$\mathbf{NPC}_1$
LOCAL	$\mathbf{P}_2$	**P**(by 2)	**P**(by 2)	**P**(by 2)



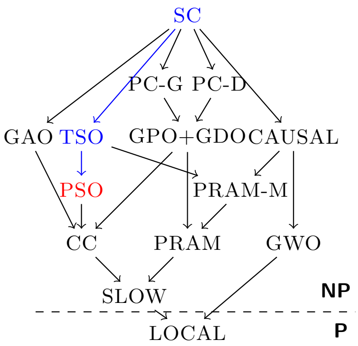
Results

Memory Model	Complexity Class of TEST(M)			
	General	Process	Length	Variables
SC	**NPC**(by 1)		**NPC**(by 3)	**NPC**(by 1)
TSO	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
PSO	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
PC-G	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
PC-D	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
GAO	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
GPO+GDO	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
Causal	**NPC**(by 1)		**NPC**(by 3)	**NPC**(by 1)
PRAM-M	**NPC**(by 1)			**NPC**(by 1)
GWO	**NPC**(by 3)		$\mathbf{NPC}_3$	
CC	**NPC**(by 1)	$\mathbf{P}_5$	$\mathbf{NPC}_6$	**NPC**(by 1)
PRAM	**NPC**(by 1)		$\mathbf{P}_4$	**NPC**(by 1)
SLOW	**NPC**(by 1)	**P**(by 5)	**P**(by 4)	$\mathbf{NPC}_1$
LOCAL	$\mathbf{P}_2$	**P**(by 2)	**P**(by 2)	**P**(by 2)



Results

Memory Model	Complexity Class of TEST(M)			
	General	Process	Length	Variables
SC	**NPC**(by 1)	**NPC**(by 7)	**NPC**(by 3)	**NPC**(by 1)
TSO	**NPC**(by 1)	**NPC**(by 7)	**NPC**(by 6)	**NPC**(by 1)
PSO	**NPC**(by 1)	$\mathbf{NPC}_7$	**NPC**(by 6)	**NPC**(by 1)
PC-G	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
PC-D	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
GAO	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
GPO+GDO	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
Causal	**NPC**(by 1)		**NPC**(by 3)	**NPC**(by 1)
PRAM-M	**NPC**(by 1)			**NPC**(by 1)
GWO	**NPC**(by 3)		$\mathbf{NPC}_3$	
CC	**NPC**(by 1)	$\mathbf{P}_5$	$\mathbf{NPC}_6$	**NPC**(by 1)
PRAM	**NPC**(by 1)		$\mathbf{P}_4$	**NPC**(by 1)
SLOW	**NPC**(by 1)	**P**(by 5)	**P**(by 4)	$\mathbf{NPC}_1$
LOCAL	$\mathbf{P}_2$	**P**(by 2)	**P**(by 2)	**P**(by 2)



Results

Memory Model	Complexity Class of TEST(M)			
	General	Process	Length	Variables
SC	**NPC**(by 1)	**NPC**(by 7)	**NPC**(by 3)	**NPC**(by 1)
TSO	**NPC**(by 1)	**NPC**(by 7)	**NPC**(by 6)	**NPC**(by 1)
PSO	**NPC**(by 1)	$\mathbf{NPC}_7$	**NPC**(by 6)	**NPC**(by 1)
PC-G	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
PC-D	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
GAO	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
GPO+GDO	**NPC**(by 1)		**NPC**(by 6)	**NPC**(by 1)
Causal	**NPC**(by 1)		**NPC**(by 3)	**NPC**(by 1)
PRAM-M	**NPC**(by 1)			**NPC**(by 1)
GWO	**NPC**(by 3)		$\mathbf{NPC}_3$	
CC	**NPC**(by 1)	$\mathbf{P}_5$	$\mathbf{NPC}_6$	**NPC**(by 1)
PRAM	**NPC**(by 1)		$\mathbf{P}_4$	**NPC**(by 1)
SLOW	**NPC**(by 1)	**P**(by 5)	**P**(by 4)	$\mathbf{NPC}_1$
LOCAL	$\mathbf{P}_2$	**P**(by 2)	**P**(by 2)	**P**(by 2)